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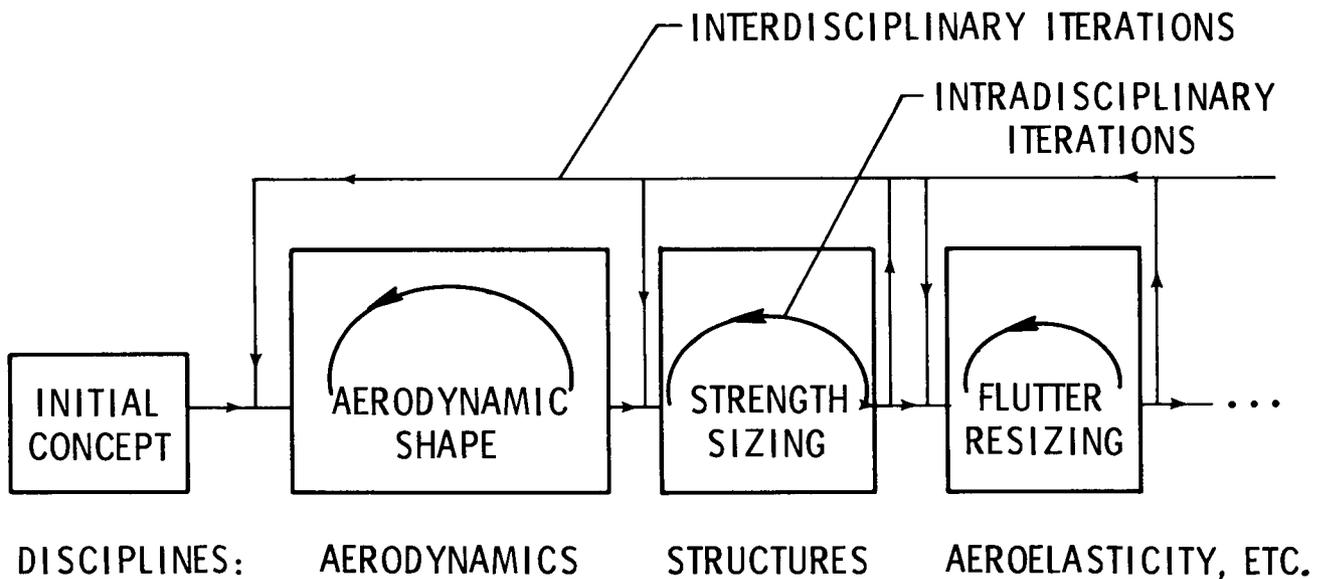
MULTIDISCIPLINARY SYSTEMS OPTIMIZATION BY LINEAR DECOMPOSITION

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In a typical design process major decisions are made sequentially. The illustrated example is for an aircraft design in which the aerodynamic shape is usually decided first, then the airframe is sized for strength and so forth. An analogous sequence could be laid out for any other major industrial product, for instance, a ship. The loops in the discipline boxes symbolize iterative design improvements carried out within the confines of a single engineering discipline, or subsystem. The loops spanning several boxes depict multidisciplinary design improvement iterations. Omitted for graphical simplicity is parallelism of the disciplinary subtasks. The parallelism is important in order to develop a broad workforce necessary to shorten the design time.

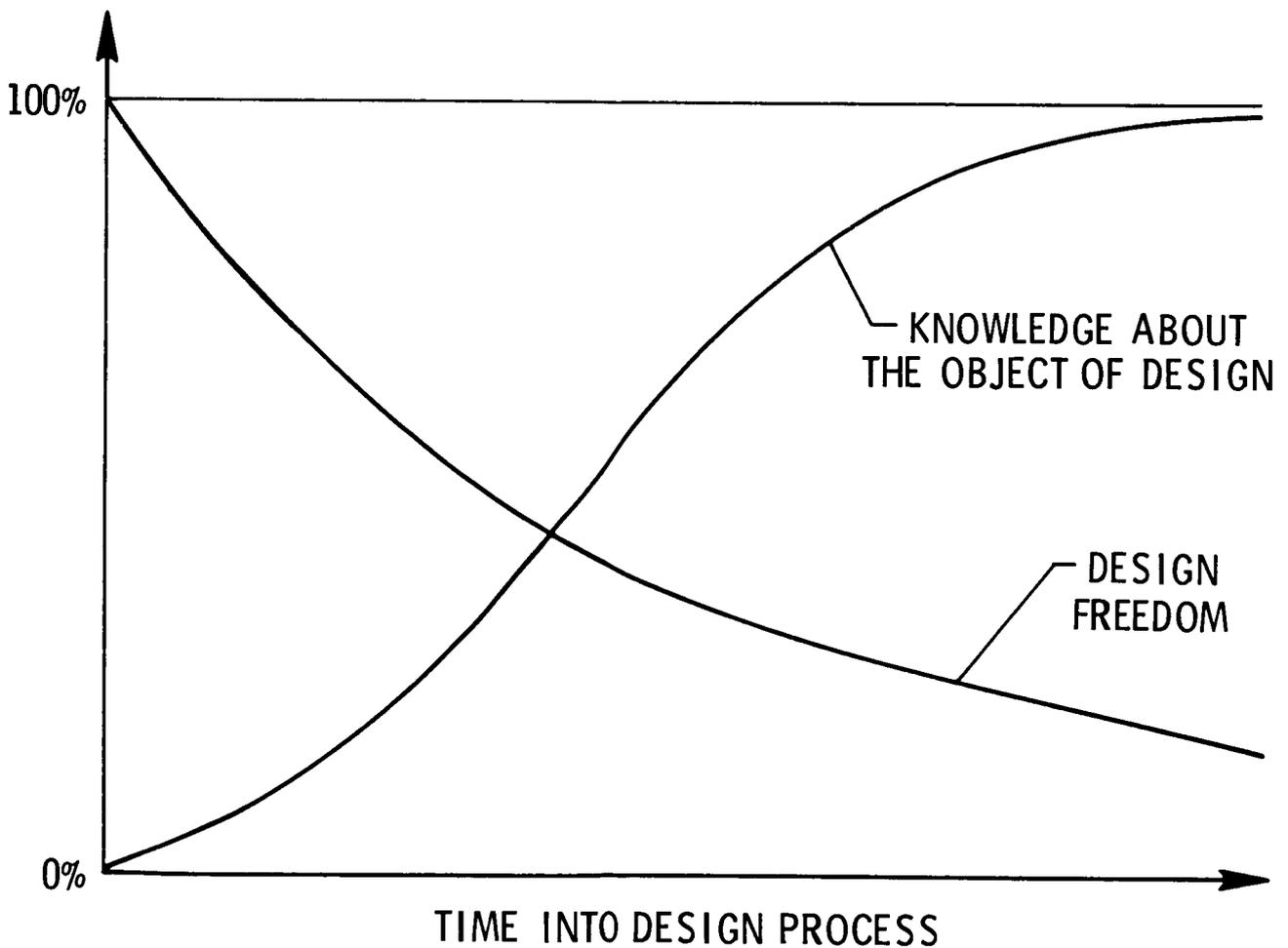
If all the intradisciplinary and interdisciplinary iterations were carried out to convergence, the process could yield a numerically optimal design. However, it usually stops short of that because of time and money limitations. This is especially true for the interdisciplinary iterations.

SEQUENTIAL DECISION-MAKING IN DESIGN PROCESS



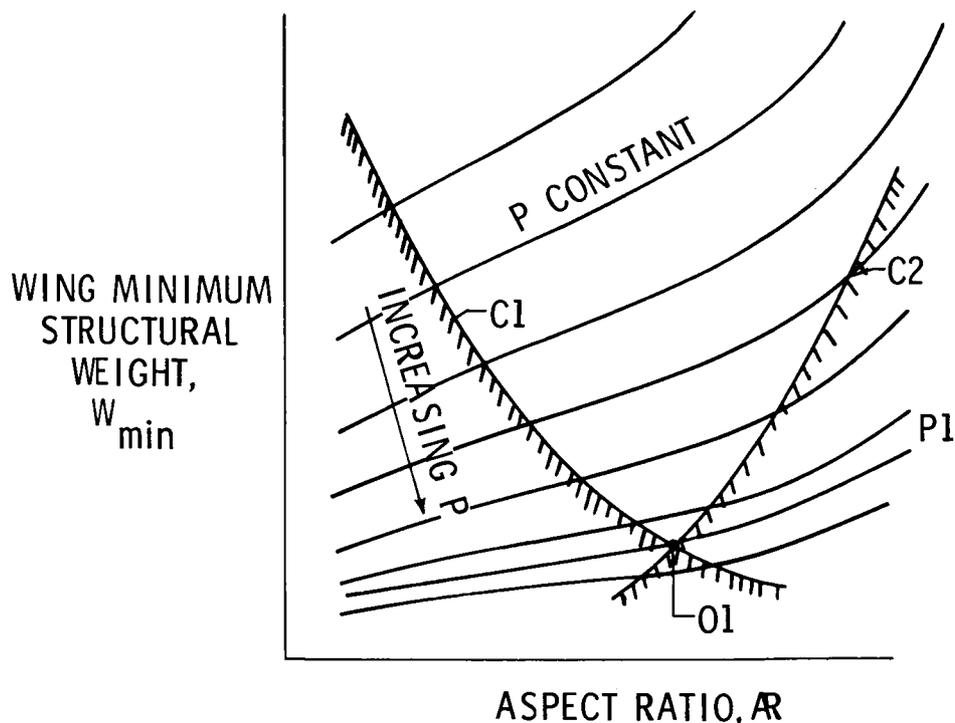
Sequential decision making leads to a paradoxical disparity between the volume of information about the object of the design and the design freedom measured by the number of design variables and options still available to the designers. The former ascends with time because of the analyses and experiments performed, while the latter declines because of casting the decisions "in concrete." The paradox is that we are gaining information but losing freedom to act on it.

PARADOX OF THE CONVENTIONAL SEQUENTIAL DECISION MAKING IN DESIGN



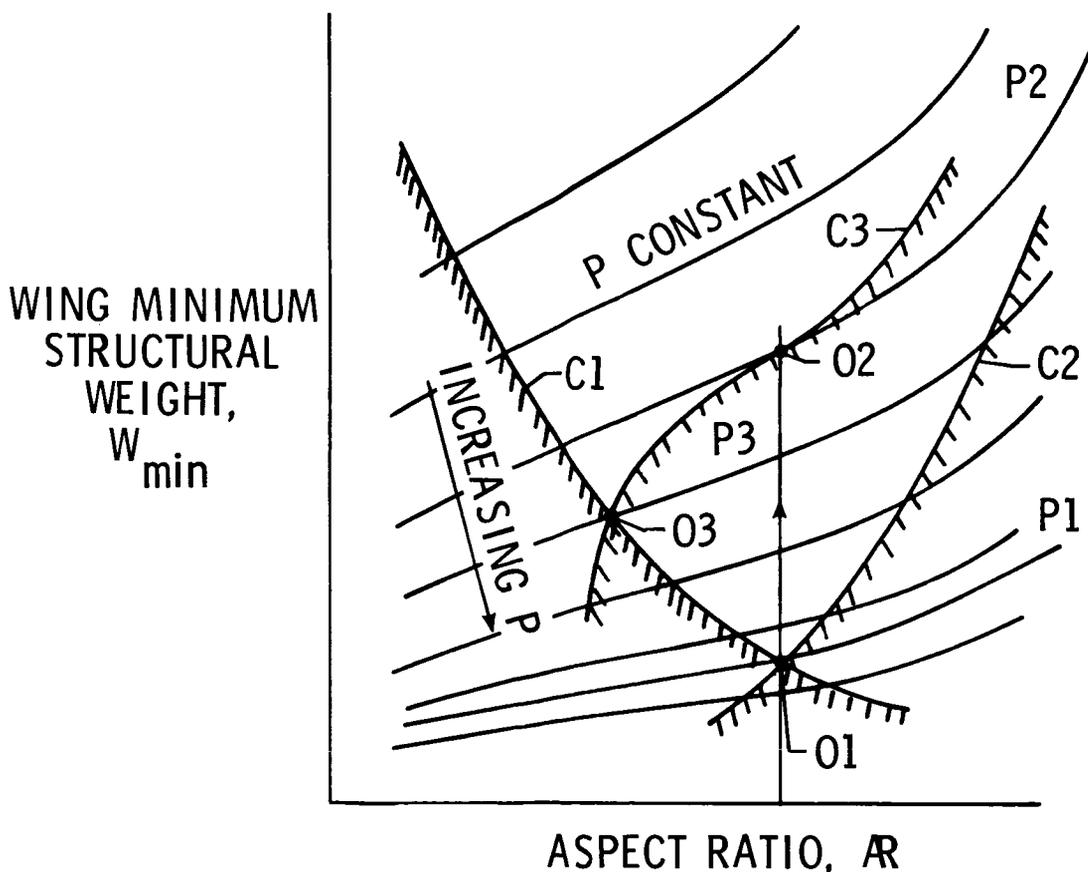
A simple example will reveal that the paradox shown in the previous chart leads to a suboptimal design. For the example, we will look into an aircraft design process at the time when wing planform and structural sizing have already been accomplished to produce a combination of two design variables, the aspect ratio and structural weight, that maximizes a measure of the aircraft performance without violating the constraints. Simplifying the example as much as possible, we can consider a design space formed by the aspect ratio and the structural weight that, we assume, has already been minimized. In that design space, shown in this chart, the aircraft performance can be depicted by a set of contour lines, each line corresponding to a constant value of the performance. Superimposed on the contour lines are the constraint curves, C1 and C2. Each constraint curve divides the design space into the feasible (constraint satisfied) and infeasible (constraint violated) subspaces (domains). The cross-hatching marks the infeasible side. It is not important for the purposes of this discussion which particular aspect of the aircraft performance was chosen as a measure of goodness (objective function) and what constraints were taken into account in plotting the set of curves, P, C1, and C2. The aircraft range for a given takeoff gross weight and payload and the wing static strength may be thought of as respective examples for P and C2. Inspection of the graphs shows that the design which maximizes P without violating C1 and C2 is at point O1.

CONSTRAINED MINIMUM FOR TWO CONSTRAINTS AND TWO DESIGN VARIABLES



Suppose now that when a flutter speed is subsequently calculated, the design O1 turns out to have too low a flutter speed - in this chart it is shown to be on the infeasible side of the flutter constraint plotted as C3. The design has to be modified to have its flutter speed raised. If at this point in the design process the configuration - the aspect ratio - is frozen, the increase of the flutter speed can be achieved by stiffening the wing structure at the price of a weight penalty by moving from O1 to O2 at a constant aspect ratio. The weight penalty reduces the performance from P1 to P2. If the configuration were not frozen, a new optimal design could be located at O3, whose performance P3, although smaller than P1, exceeds P2 ($P2 < P3 < P1$). The difference $P3 - P2$ is a performance penalty for the sequential freezing out of the design options in a sequential design process. We can say that design O2 is suboptimal relative to the design O3. Another look at this and the preceding chart, and a little reflection, will show that although the magnitude of the performance penalty, $P3 - P2$, depends on the shape of the functions involved ($P, C1, C2, C3$), its existence does not. Consequently, the example reveals that suboptimal results can be expected in a sequential design process in which each additional stage restricts the number of design variables while bringing in new constraint violations that must be removed.

A NEW CONSTRAINT ADDED

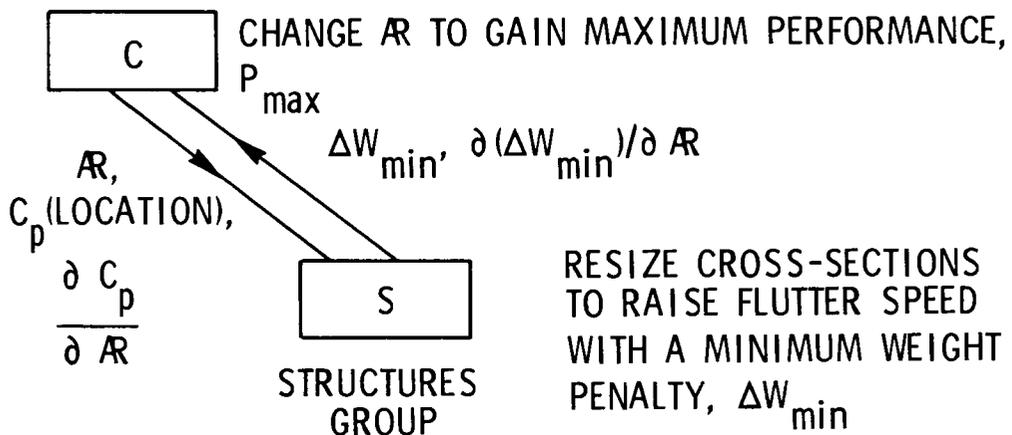


To demonstrate an alternative based on a system approach, reenter the example at the point where the flutter deficiency of the design, labeled 01 in the preceding chart, has been found. The essence of the system approach is decomposition of a large problem into several smaller ones without losing the coupling. Therefore, we recognize in this case that two engineers, or engineering groups, must fix the flutter problem with the least penalty to the performance, P , by cooperating and yet each doing a separate subtask. In this chart the individuals, or groups, are labeled C - for configuration, including aerodynamics and performance, and S - for structures.

The subtask of correcting the flutter problem with a minimum weight penalty ΔW_{\min} is carried out by S for a particular aspect ratio set constant, but only temporarily, by C, and for aerodynamic analysis results (e.g., pressure distribution) and their sensitivity to aspect ratio - all supplied by C. The result produced by S is a flutter-free design at a minimum weight penalty, along with the sensitivity of that design to aspect ratio. That sensitivity is quantified in the form of derivatives of the weight penalty and cross-sectional dimensions with respect to the aspect ratio.

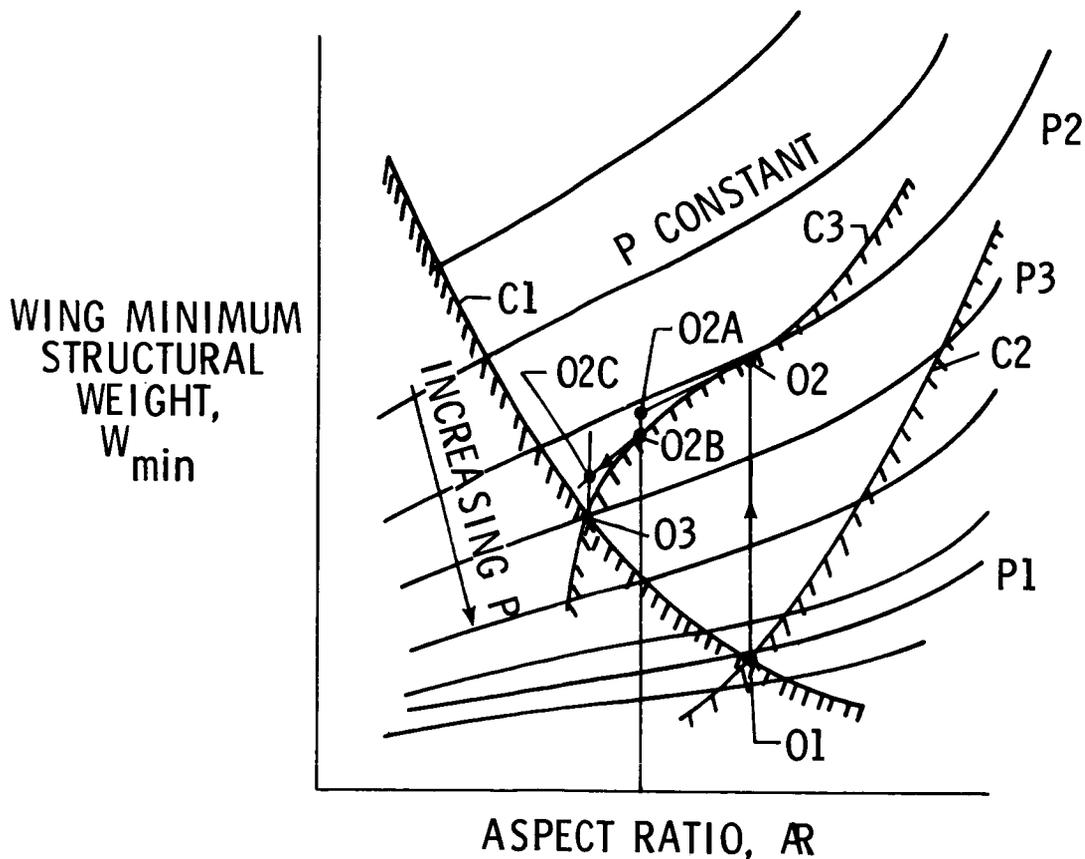
FINDING NEW CONSTRAINED MINIMUM BY ALTERNATING BETWEEN TWO ENGINEERING DISCIPLINES

CONFIGURATION GROUP



Completion of the above task moves the design from O1 to O2 in this chart, exactly as in the previous discussion. However, group C will now recover a part of the performance penalty by changing the aspect ratio and the weight penalty simultaneously. In this operation, the weight penalty is not an independent variable but is tied to the aspect ratio variation by the sensitivity derivative which tells how much the weight penalty must change per unit of aspect ratio variation to keep the flutter constraint satisfied. Such dependence of weight penalty on aspect ratio is only a linear approximation of a true nonlinear relation and can be depicted by the tangent to C3 at O2 shown in this chart. The configuration improvement produced by C calls now for a move along that tangent toward the increasing performance; that is, toward O3. The move should stop when the tangent veers off too far from C3 in order to let group S repeat its subtask to recover from the linearization error by regenerating the minimum weight penalty and its sensitivity derivative at the new value of the aspect ratio. Thus, by alternating subtasks performed by C and S we can improve the design by moving toward the theoretical optimum at O3 in a staircase fashion: O2 to O2A, to O2B, to O2C, and so on, as long as we see that the performance improvement is worth the effort.

A PATH TOWARD NEW CONSTRAINED MINIMUM



Having introduced the idea of decomposition by means of a simple example we will now generalize the four objectives shown in the chart as guidelines. The first three are self evident. The last one deals with the disparity between the large volume of information that is being processed within a subtask and a relatively small volume of information that couples the subtask (subsystem) to other subtasks (subsystems). For example, contrast the mass of data being manipulated in a finite element analysis of an airframe with the input data of loads, mechanical properties, and geometry, and with the structural weight and critical constraint data which is all that is fed back to the aircraft performance analysis. The decomposition scheme should exploit that disparity. Lack of such disparity indicates that either the decomposition scheme is improper or the problem is not decomposable.

DECOMPOSITION OBJECTIVES

- **BREAK LARGE TASK INTO A NUMBER OF SMALLER ONES**
- **PRESERVE THE COUPLINGS AMONG THE SUBTASKS**
- **EXPLOIT PARALLELISM TO DEVELOP A BROAD WORKFRONT OF PEOPLE AND COMPUTERS**
- **EXPLOIT THE DIFFERENCE OF VOLUME BETWEEN A RELATIVELY LARGE AMOUNT OF INFORMATION PROCESSED INTERNALLY IN A SUBTASK AND A RELATIVELY SMALL VOLUME OF THE COUPLING INFORMATION**

There are several decomposition procedures in the literature (ref. 1). However, a literature survey failed to reveal a method that would be capable of accounting for the couplings among the system and subsystems without having to reoptimize the subsystems for every variation of the parent system design variables and that would apply to nonlinear programming problems. Since such repeated reoptimizations would be cost prohibitive in most large-scale engineering applications, a new approach that accounts for the system-subsystem couplings without the repetitive subsystem reoptimizations has been developed at Langley Research Center and is now at a stage of testing and verification. The approach is called "a linear decomposition" for reasons that will become apparent soon.

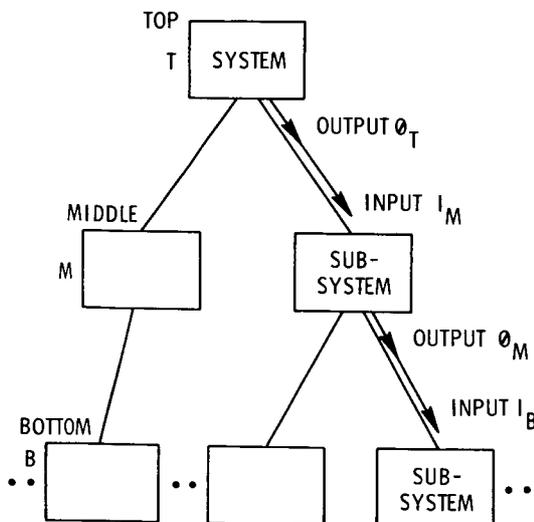
SEVERAL WAYS TO HANDLE THE COUPLINGS IN A DECOMPOSED SYSTEM

- BODY OF LITERATURE
- THE PROPOSED APPROACH: A LINEAR DECOMPOSITION

For generality, the chart shows a generic system decomposed into subsystems that form a hierarchical, three-level tree. If the system were a structure, the top level would represent the assembled structure, each subsystem at the middle level would correspond to a substructure, and the bottom level subsystems would simulate individual structural components (e.g., stiffened panels). Thus, three levels is the minimum we need to have each level qualitatively different for generality of the discussion. We assume that the system has been initialized so that physical characteristics are completely defined at each level. It is not necessary for the initialized system to be feasible. The analysis proceeds from top to bottom so that output from analysis of a parent subsystem becomes input for analysis of the subordinated subsystems. For an example, consider a structure assembled of substructures and loaded by forces applied at the substructure boundary nodes. The substructure boundary forces from the assembled structure analysis are fed into the substructure analysis as loads, and the internal forces from substructure analysis enter into the individual structural component analysis.

In many engineering applications, the decomposition must account for the fact that inputs to analysis of a given subsystem may be coming not only from its parent but from any other subsystem at the same or even a different level, including inputs from the subordinated subsystems to their parent. An example of the latter can be drawn from the substructuring analysis in the case where the loads applied to a substructure interior nodes are reduced to the loads applied at the substructure boundary nodes, by performing analysis at the substructure level before commencing the assembled structure analysis. In other words, a system decomposition may lead to a network rather than the "top-down" graph shown in the chart. However, we will limit this discussion to the case depicted in the chart in order to keep it as simple as possible for a clear introduction of the basic approach. Extension of the approach necessary to handle the network systems is presented in ref. 2. It is important that analyses at each level include sensitivity analysis to produce derivatives of the output quantities with respect to the input quantities. These derivatives measure sensitivity of behavior (response). Obviously, if there are several subsystems at a given level, they can be analyzed concurrently.

ANALYSIS



- INITIALIZATION
- TOP-DOWN ANALYSIS
EACH SUBSYSTEM RECEIVES
INPUTS FROM ITS PARENT,
UNDERGOES ANALYSIS AND
SENDS OUTPUT TO SUB-
SYSTEMS BELOW

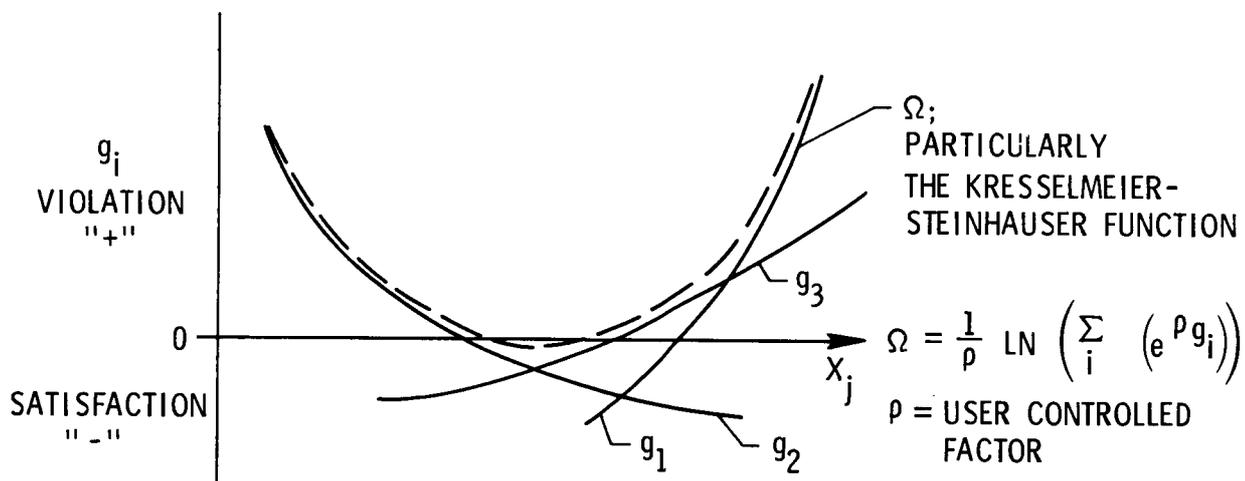
This chart introduces a cumulative constraint that will be needed in further discussion. The cumulative constraint is a single number that measures the degree of satisfaction, or violation, of an entire set of constraints. There are several ways to formulate the cumulative constraint as a function of the constraints in the set, for instance, the well known quadratic exterior penalty function is a cumulative constraint. The particular formulation adopted here is a function shown by the equation below and referred to as the Kresselmeier-Steinhauser function. The function is continuous and differentiable, in contrast to the envelope of the constraint functions, which is slope discontinuous at the constraint function intersections, and, as seen in the graph, follows the constraint envelope at a distance that is user-controlled by the factor ρ . Increase of the factor draws the function closer to the envelope. The factor ought to be set so that the cumulative constraint function does not lose numerical differentiability by forming sharp "knees" at the constraint intersections.

CUMULATIVE CONSTRAINT

- A SINGLE NUMBER MEASURE OF THE DEGREE OF SATISFACTION, OR VIOLATION, FOR A SET OF CONSTRAINTS

$$\Omega = f(g_i) \quad i = 1 \rightarrow m; \quad g_i = \frac{\text{DEMAND}}{\text{CAPACITY}} - 1 \leq 0$$

- AN APPROXIMATE ENVELOPE FUNCTION

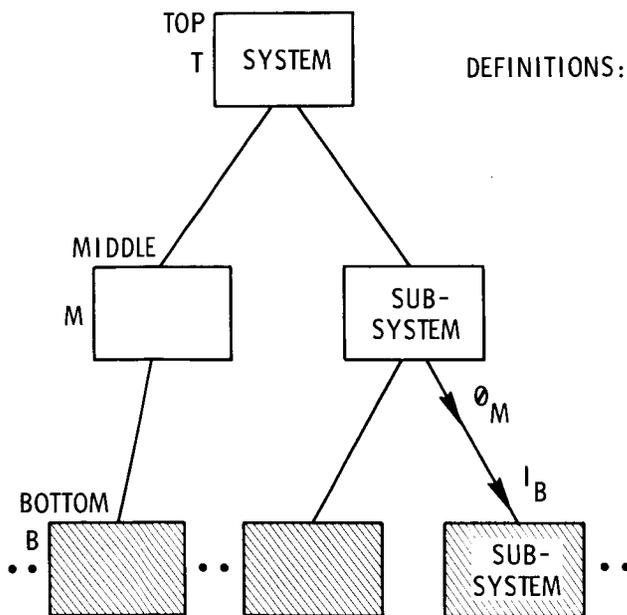


Having completed the analysis and introduced the cumulative constraint, we now begin the optimization which will proceed from the bottom up. Each subsystem optimization at the bottom level is characterized as follows:

1. design variables: physical quantities local to the subsystem, e.g., detailed cross-sectional dimensions of a panel
2. objective function: the cumulative constraint of the subsystem constraints such as local buckling, stress, etc
3. inequality constraints: upper and lower limits on the design variables
4. constant parameters: inputs received from the parent subsystem
5. equality constraints: these constraints may be required in order to preserve the constancy of the parameters (for example, if a parameter is a total cross-sectional area of a panel, an equality constraint on the detailed cross-sectional dimension variables is needed)

The use of a cumulative constraint as the subsystem objective is a logical choice because it is a non-dimensional quantity and therefore comparable among the subsystems regardless of their physical nature, which may vary from one to another. The subsystem optimization is followed by sensitivity analysis of the minimum of the objective with respect to the subsystem input quantities (equal to the output from the parent subsystem). This analysis is called optimum sensitivity analysis to distinguish it from the behavior sensitivity analysis and is carried out not by finite difference but by a special algorithm described in ref. 3. Thus, the results from each subsystem optimization are the minimum of the cumulative constraint and its sensitivity to the output from the parent subsystem. These results are now carried upward to the parent subsystem. If there are several subsystems at a given level, their optimizations can be executed concurrently.

OPTIMIZATION: BOTTOM LEVEL



DEFINITIONS: DESIGN VARIABLES = X_B
 INPUT = I_B RECEIVED FROM ABOVE
 $I_B \equiv \theta_M$

OBJECTIVE FUNCTION:
 CUMULATIVE CONSTRAINT
 Ω^B

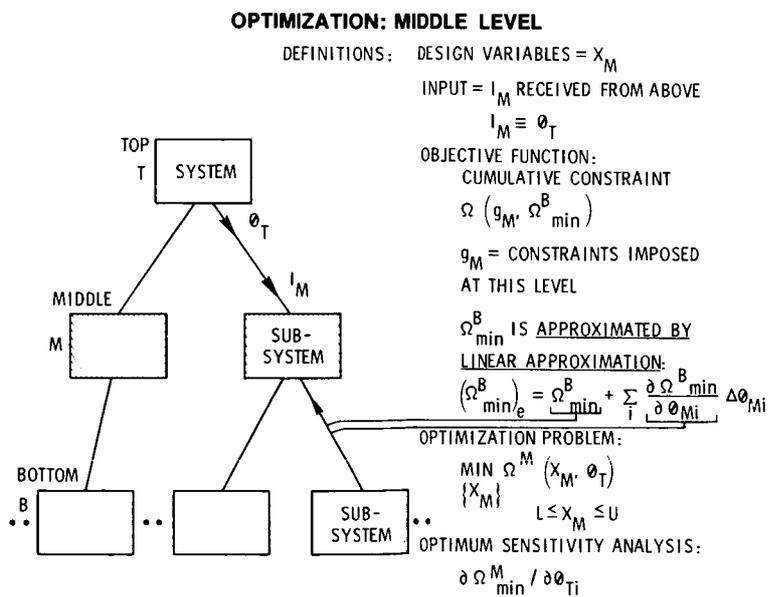
OPTIMIZATION PROBLEM:
 $\text{MIN } \Omega^B (X_B, \theta_M)$
 $\{X_B\}$
 $L \leq X_B \leq U$

OPTIMUM SENSITIVITY ANALYSIS:
 $\partial \Omega_{\min}^B / \partial \theta_{Mi}$

Now, moving up one level to the middle level, we perform a subsystem optimization for each subsystem at that level. The optimization is formulated as follows:

- (1) Design variables: physical quantities local to the subsystem, e.g., membrane stiffness of the wing box at several locations over the wing
- (2) Objective function: cumulative constraint for a set of constraints that includes the constraints intrinsic to the subsystem itself (e.g., limit on the wing tip deflection) and the minimum values of the cumulative constraints transmitted from the subordinated lower level subsystems (These minimum values are estimated by linear extrapolation (see equation) as a function of the middle level subsystem design variables by means of the optimum sensitivity derivatives taken with respect to the subsystem output quantities which, in turn, are governed by the subsystem design variables. This linear extrapolation eliminates the need to reoptimize the subordinated subsystems for each design variable variation introduced in the parent subsystem and gives the method its name of the linear decomposition.)
- (3) Constant parameters
- (4) Inequality constraints
- (5) Equality constraints are analogous to those defined for the bottom level

The results are a minimum of the cumulative constraint and its derivatives with respect to the system output. They are now carried to the top level.



Optimization at the top level involves:

1. design variables that govern the entire system, for an aircraft example: configuration geometry, structural weight prescribed for the airframe, etc.
2. objective function as a measure of the system performance, e.g., fuel consumption or Direct Operating Cost
3. inequality constraints on the system performance, e.g., take-off field length, the upper and lower limits on the design variables, and the cumulative constraints from each subsystem linearly extrapolated by means of the optimum sensitivity derivatives (These constraints also include the side constraints and move limits to control the linear extrapolation error.)

Thus, the top level optimization deals with the system performance directly, and has embedded in it the approximation to all the subsystem constraints in the form of the linear extrapolation based on the subsystem optimum sensitivity derivatives. These derivatives quantify the design trade-offs among the subsystem and account for their couplings.

The top-down analysis and the bottom-up optimizations constitute one cycle of the iterative procedure which continues until the extremum of the system objective is found and all the system constraints and the subsystem cumulative constraints are satisfied. For more algorithmic detail, one may consult ref. 2.

OPTIMIZATION: TOP LEVEL

DEFINITIONS: DESIGN VARIABLES = X_T

OBJECTIVE FUNCTION: A MEASURE $F(X_T)$ OF THE SYSTEM PERFORMANCE

CONSTRAINTS:

$g_T =$ SYSTEM PERFORMANCE CONSTRAINTS

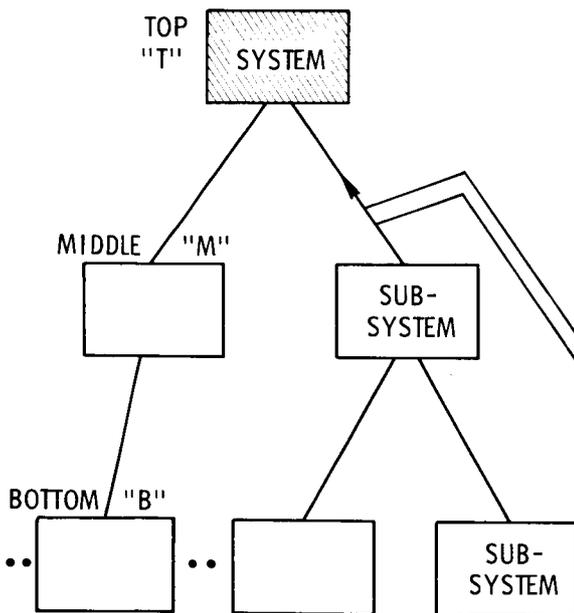
$\Omega_{min}^M \leq 0$ FOR EACH M-LEVEL SUBSYSTEM

OPTIMIZATION PROBLEM:

$$\begin{aligned} & \text{MIN } F(X_T) \\ & \{X_T\} \\ & g_T \leq 0 \\ & (\Omega_{min}^M)_e \leq 0 \\ & \dots \\ & L \leq X_T \leq U \end{aligned}$$

WHERE:

$$(\Omega_{min}^M)_e = \Omega_{min}^M + \sum_i \frac{\partial \Omega_{min}^M}{\partial \theta_{Ti}} \Delta \theta_{Ti}$$



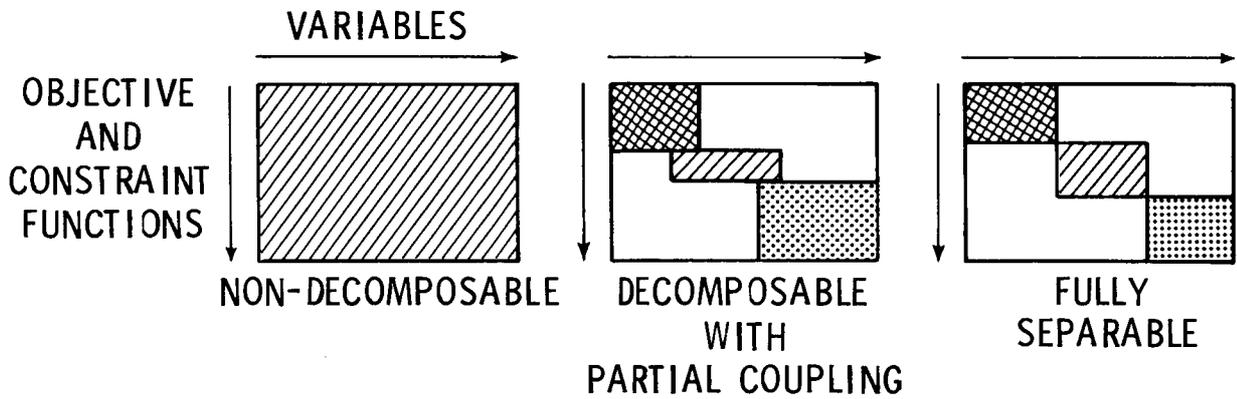
While the procedure described in the previous five charts is generic, the decomposition of the system is problem dependent. It can be done by a common sense inspection and judgment. It can also be done formally by a matrix of the design variables listed along the top and the objective and constraint functions listed vertically. A dot at the row-column intersection means that the variable corresponding to the column appears in the equation corresponding to the row, and a blank means that the variable does not appear in that equation. The three examples show typical patterns.

MANY WAYS TO DECOMPOSE A SYSTEM

- HEURISTIC: BY EXAMINATION OF THE SYSTEM PHYSICAL MAKE-UP.

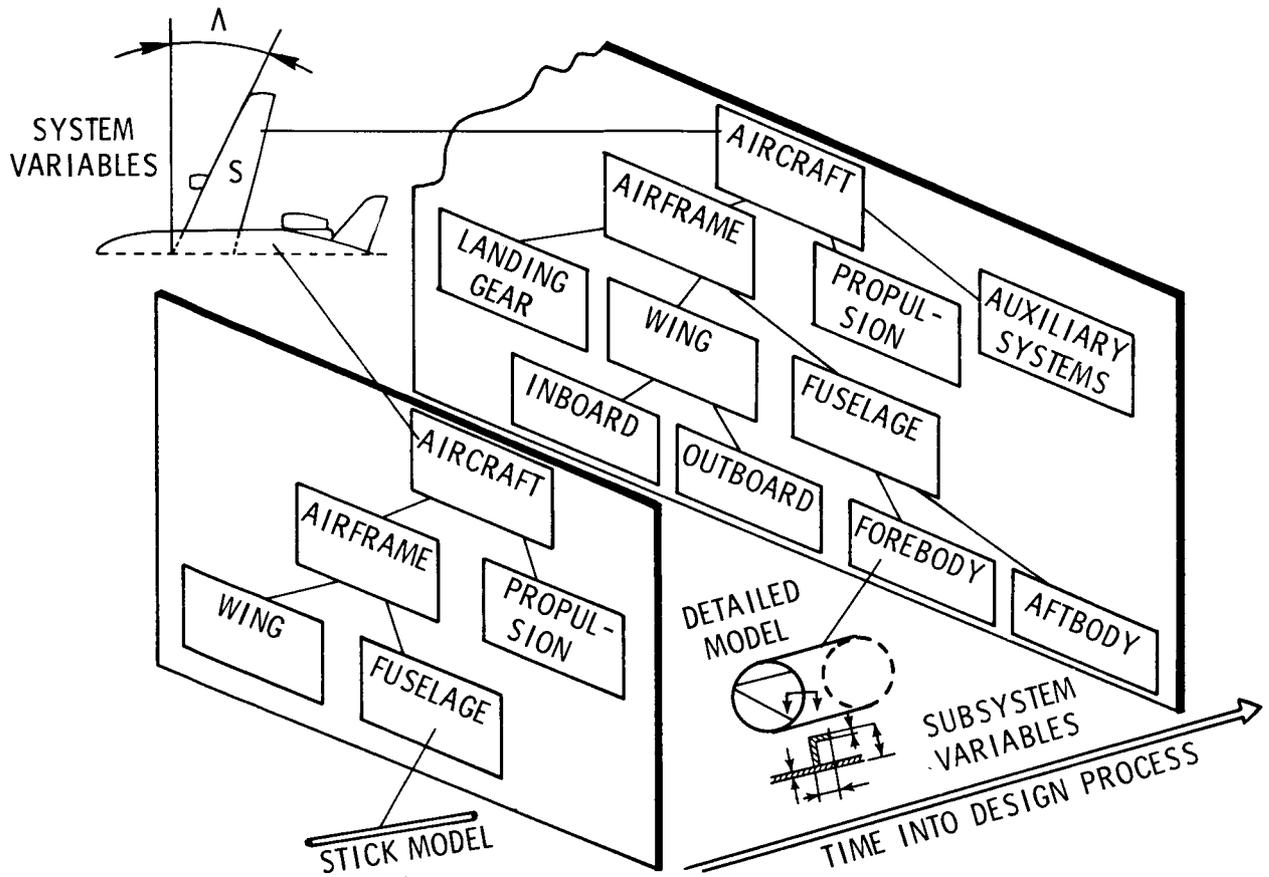


- FORMAL: BY INSPECTION OF THE FUNCTIONAL RELATIONS THAT GOVERN THE PROBLEM



Once the decomposition tree is established, it can be grown with respect to the number of subsystems and the depth and detail of analysis. This adaptability permits maintaining the same overall logic of approach at various stages of design, while changing the modules in that logic - a desirable feature from the standpoint of the process integration.

DECOMPOSITION ADAPTS TO DESIGN STAGE



The multilevel procedure described here is still being developed toward a state of maturity required for industrial applications. To achieve that state, research continues to investigate the issues listed below.

ISSUES TO BE INVESTIGATED

- CONVERGENCE: OVERALL, LOCAL
- COMPUTATIONAL COST
- LATERAL AND REVERSE COUPLINGS
- ACCURACY OF LINEAR EXTRAPOLATIONS BASED ON SENSITIVITY
- SUBTASK SYNCHRONIZATION
- CONSISTENCY OF THE ANALYSIS LEVELS IN VARIOUS SUBTASKS
- JUDGMENTAL DECISIONS, INCLUDING DISCRETE DECISIONS, AND HUMAN CONTROL

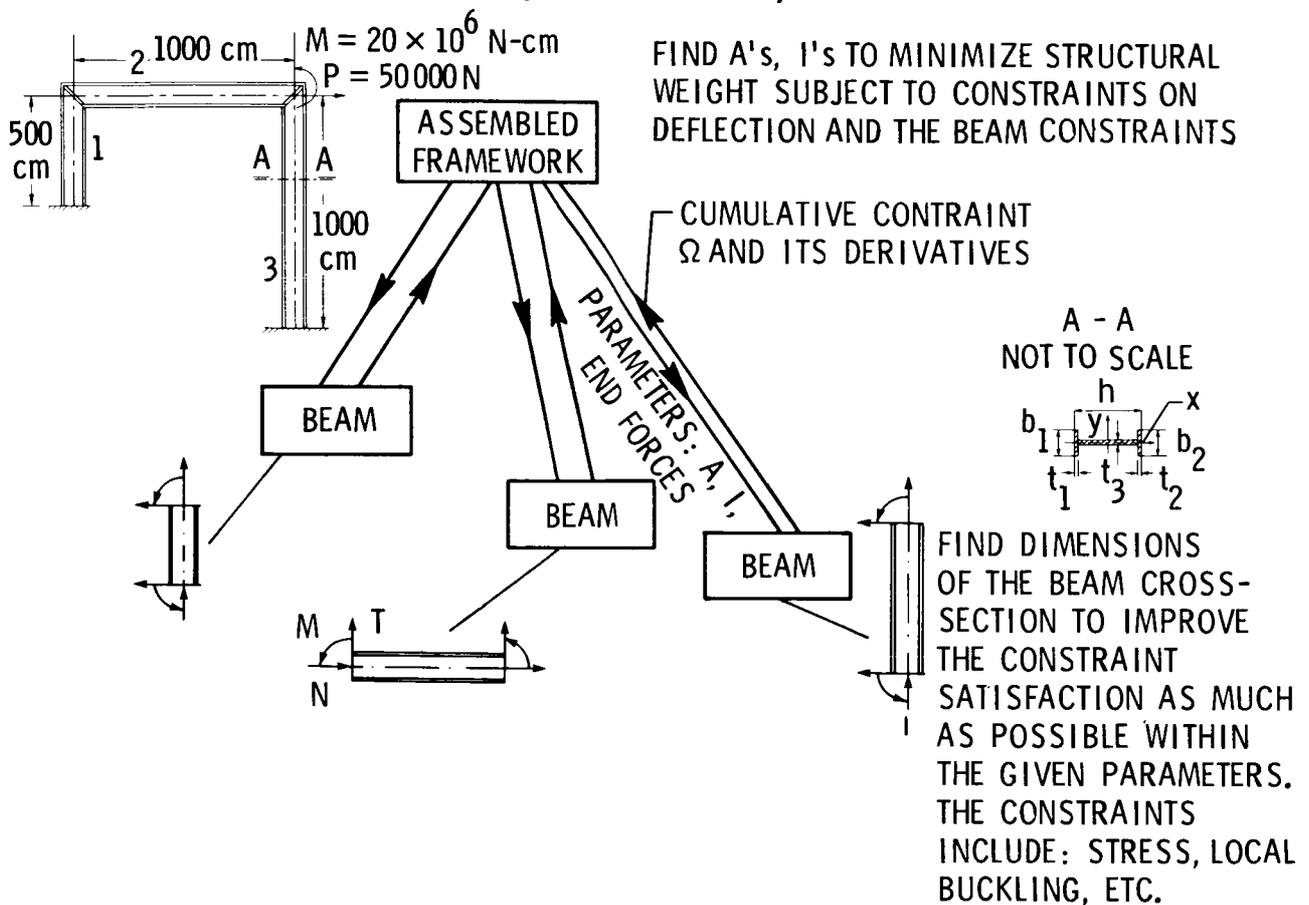
The development toward maturity involves a literature survey, numerous tests of several variations of the algorithm using very simple test cases, and a fairly large structural test case. A multidisciplinary test is under way for reconfiguration of a transport aircraft wing treated as a part of an aircraft system and, also, a wing separated from the aircraft. The issues of computational parallelism and synchronization among the subtasks are being explored using a network of microcomputers connected to a central hard disk.

RESEARCH INTO THE MATHEMATICS OF MULTILEVEL DECOMPOSITION

- GOAL: A LEVEL OF MATURITY REQUIRED FOR INDUSTRIAL APPLICATIONS
- LINES OF RESEARCH:
 - SURVEY OF LITERATURE
 - SMALL TEST CASE - A SIMULATOR - TO TEST VARIOUS ALGORITHMS
 - APPLICATION TEST CASES: STRUCTURES
LOCKHEED PROJECT
ISOLATED WING CASE
PARALLEL COMPUTING
(A NETWORK OF APPLES)
 - GRANT ACTIVITIES TO BE REPORTED IN THIS SESSION

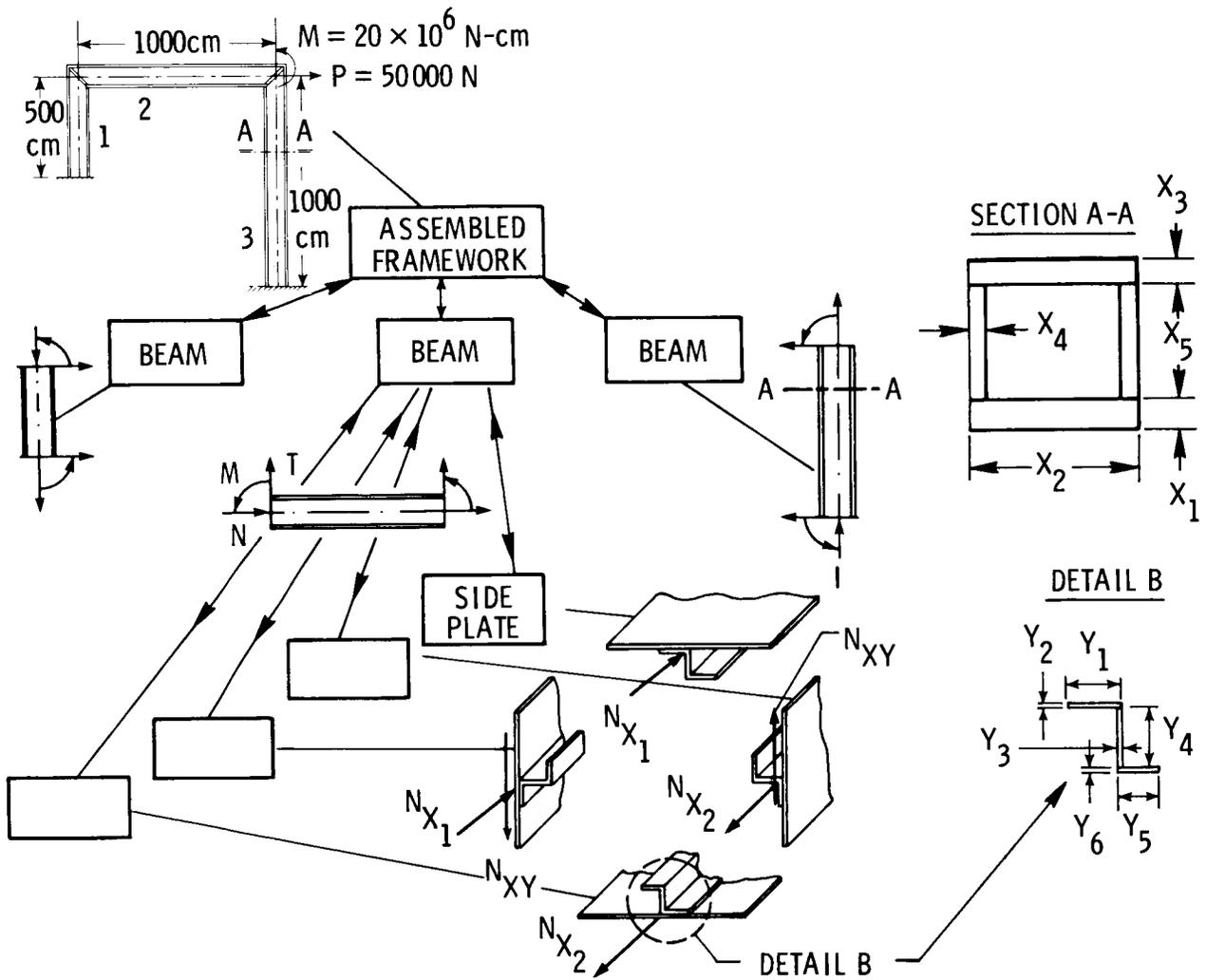
A two-level structural optimization of a framework has been successfully carried out and reported in ref. 4. The decomposition in this case exploits the fact that the end forces acting on each I-beam in the framework can be calculated using A and I for the beams without directly using the beam cross-section design variables. Furthermore, the local constraints in a beam can be calculated using the beam's detailed cross-sectional dimensions and the end forces. Thus, the A 's and I 's are the system design variables and the detailed dimensions are the subsystem design variables. The beam is optimized by reducing the cumulative constraint to a minimum (maximizing the safety margin). In the process, the beam cross-section is reportioned while preserving the A and I prescribed for the beam at the system level.

MULTILEVEL OPTIMIZATION: A FRAMEWORK TEST STRUCTURE (TWO LEVELS)



The two-level framework structure has been extended to three levels by replacing the I-beams with the box beams made up of stringer-reinforced panels. The panels add the third, bottom level of subsystems. This makes the test more general because it now contains all three level categories: top, middle, and bottom. At the time of this writing the tests are still in progress.

MULTILEVEL OPTIMIZATION: A FRAMEWORK STRUCTURE (THREE LEVELS)



This is a summary of multilevel structural optimization development and testing. The method was also applied in a test case of a high-performance sailplane wing design (ref. 5). The results obtained to date are encouraging.

TEST APPLICATIONS IN STRUCTURES

TWO LEVELS: GOOD CORRELATION WITH A SINGLE LEVEL TEST CASE

- MINIMUM WEIGHT AGREED WITHIN 2%
- QUITE LARGE DIFFERENCES IN OPTIMUM DESIGN VARIABLES = A "SHALLOW" OPTIMUM

(ref. 4)

SAILPLANE WING (ref. 5)

THREE LEVELS: REFERENCE SINGLE LEVEL TEST CASE ESTABLISHED

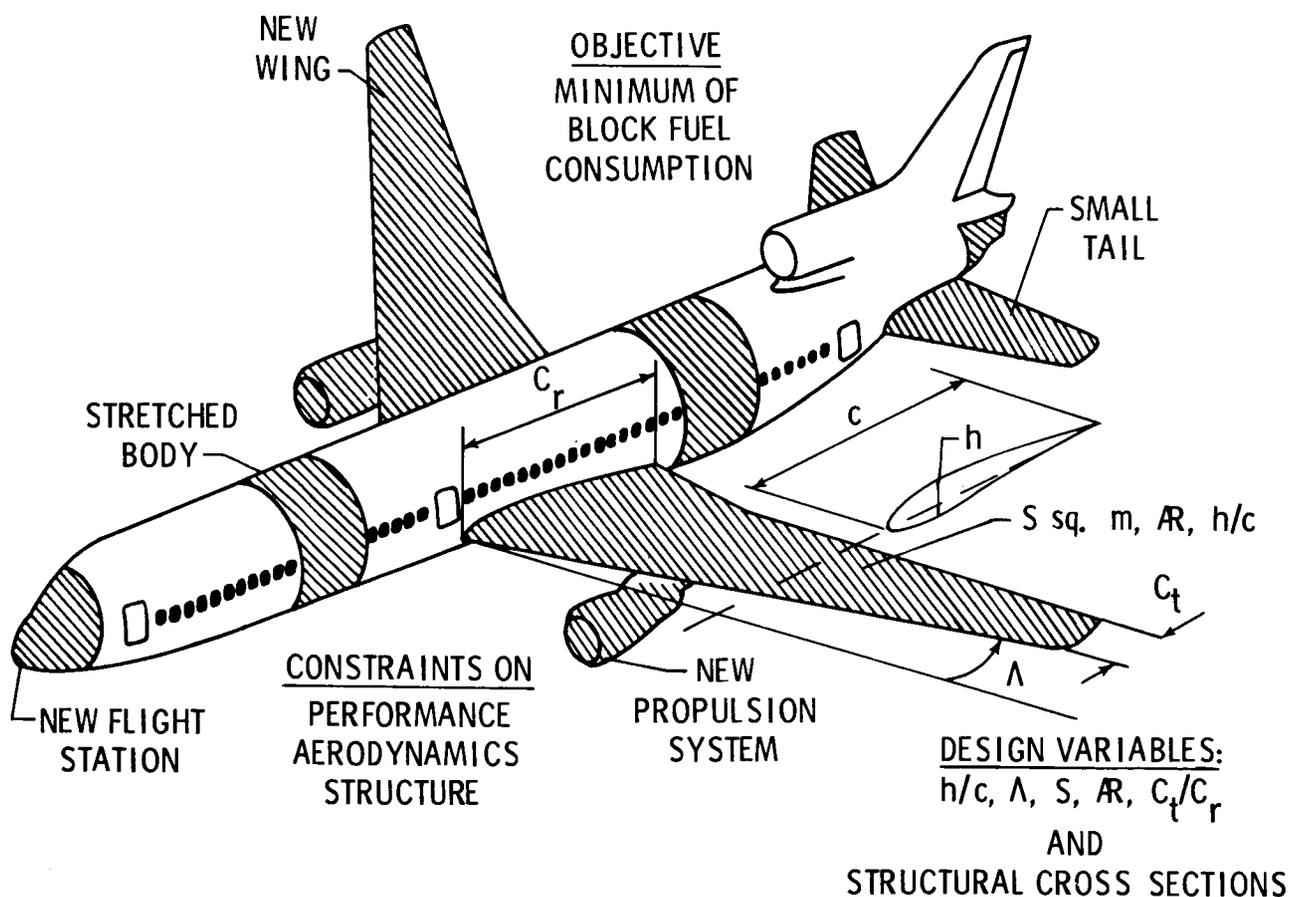
THREE-LEVEL PROCEDURE IMPLEMENTED AND DEBUGGED

RESULTS BEING GENERATED FOR WORK-IN-PROGRESS

(ref. 4)

The wing of a transport aircraft (Lockheed L-1011) is to be reconfigured to minimize fuel consumption for a given mission. The design variables are selected from the geometrical configuration dimensions noted in the drawing and the detailed structural dimensions (not shown) of the wing cover panels reinforced by stringers. A long list of constraints includes the local effects, such as local buckling, and the system performance constraints, such as takeoff field length. The testing is being done jointly with Lockheed-California which supplies the mathematical model (e.g., a finite element model) and the mission and load data, and reconfigures the wing by means of parametric studies using the state-of-the-art tools in each discipline. The Langley team is using the linear decomposition. Comparison of the results will allow assessment of the relative merits of the proposed method. Further details of the test are provided in reference 6.

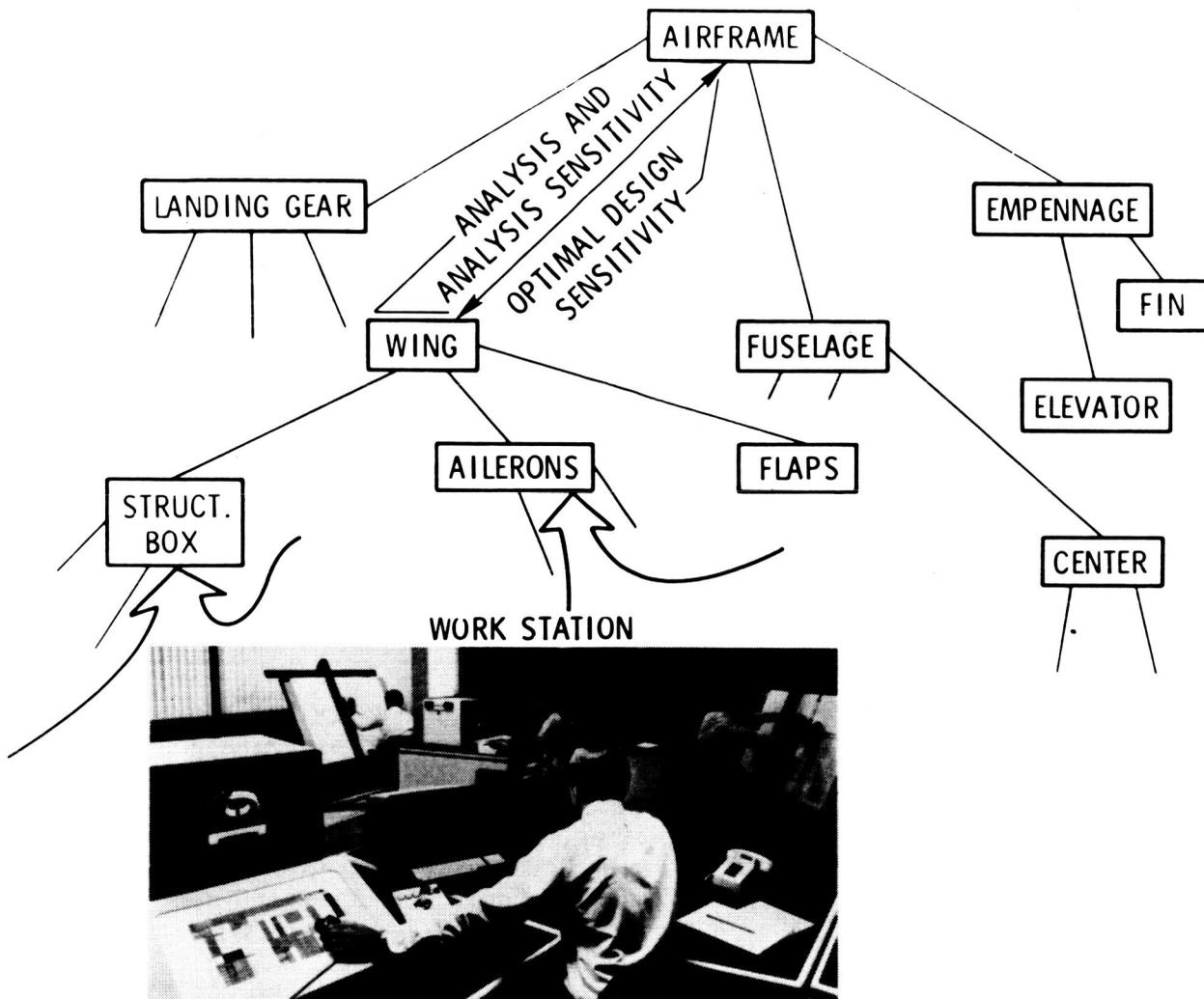
**MULTILEVEL OPTIMIZATION APPLICATION
A COOPERATIVE VENTURE WITH LOCKHEED-CALIFORNIA
(LOCKHEED-GEORGIA INVOLVED)**



**ORIGINAL PAGE IS
OF POOR QUALITY**

Implementation of the proposed multilevel, linear decomposition in design will fit the existing organization of professionals and will exploit the new technology of distributed, parallel computing. It will introduce the new element of mathematical quantification of the design trade-offs and will establish a precise definition for information exchange among the specialists working on their subtasks. Under the proposed scheme, the decision makers at each level will know the consequences of their decisions on other coupled subsystems. Based on this knowledge, it should be possible to improve the design integration toward higher performance and lower cost.

DESIGN OFFICE ORGANIZATION



CONCLUSIONS

- THEORY OF ONE PARTICULAR APPROACH TO DECOMPOSITION DOCUMENTED (ref. 2)
- TESTS ON STRUCTURES: FRAMEWORK, SAILPLANE WING
- TEST ON AN AIRCRAFT CONFIGURATION UNDER WAY
- RESEARCH AND DEVELOPMENT CONTINUE TOWARD MATURITY REQUIRED FOR INDUSTRIAL APPLICATIONS

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